Math 308, Bridge to Advanced Math, Prof. Santoro

Chapter 3 Homework Solution

5) Note that for any real x, $(x-1)^2 \geq 0$. Expanding, we obtain that $x^2+1 \geq 2x$, and if $x \neq 0$, we have

$$x + \frac{1}{x} \geq 2.$$

Therefore, the implication is true vacuously.

6) If a, b and c are odd integers, then there exist $k, \ell, m \in \mathbb{Z}$ such that $a = 2k+1$, $b = 2\ell+1$ and $c = 2m+1$. Hence,

$$abc = (2k+1)(2\ell+1)(2m+1) = 2((2\ell+1)(k(2m+1)+m)+\ell)+1,$$

an odd number.

Since zero is even, the hypothesis is always false, and hence the statement is true vacuously.

15) Note that $A \cap B = \{3, 5, 7, 9\}$. The proof is done by testing the statement for each element.

24) Since n is an integer, $\cos(n\pi/2)$ can only be one of $-1, 0, 1$. If it is even (hence zero), then $\frac{n\pi}{2} = \frac{\pi}{2} + \pi k$, for some integer k. In other words, $n = 2k+1$, that is, n is odd. This implies that $2n^2+n$ is also odd.

For the converse statement, we prove it by contrapositive: assume $\cos(n\pi/2)$ is odd, hence ± 1. Hence, $\frac{n\pi}{2} = \pi\ell$ for some integer ℓ, and so $n = 2k$. This implies that $2n^2+n$ is even.

29) (Done in class) If ab is odd, then both a and b are odd. Hence, there exist integers k and ℓ such that $a = 2k+1$ and $b = 2\ell+1$.

Therefore, $a^2+b^2 = 2(2k^2+2k+2\ell^2+2\ell+1)$, which is an even integer.