Math 190 – College Algebra and Trigonometry (Prof. Santoro) Exam 2, November 3, 2015

NAME: Solutions

Instructions: Please read each question carefully, show all work, and check afterwards that you have answered all of each question correctly.

Important: No books, calculators, or notes are allowed. Turn off cell phones, alarms, and anything else that makes noises! You must show all your work to receive credit. Any crossed out work will be disregarded (even if correct). Write one clear answer with a coherent derivation for each question. All questions are worth the same number of points. Good luck!

1. Find the diameter and the center of the circle $x^2 + y^2 + 4x - 8y = 0$.

We complete the squares: we need to add $(4/2)^2$ and $(8/2)^2$ to both sides of the equation, and regroup alike terms:

$$(x2 + 4x + 4) + (y2 - 8y + 16) = 0 + 4 + 16.$$

Factoring, we obtain the equation

$$(x+2)^2 + (y-4)^2 = 20$$

and from this we can read that the center is (-2, 4), and the radius is $\sqrt{20} = 2\sqrt{5}$. Since the diameter is twice the radius, we find that the diameter is $4\sqrt{5}$.

2. Find the equation of the line passing through the point P(1,3), and perpendicular to the line x + 2y = 7.

The line we want is perpendicular to the given line x + 2y = 7. Rewriting this equation, we obtain $y = -\frac{1}{2}x + \frac{7}{2}$. This line has slope $-\frac{1}{2}$, which tells us that the line we want, perpendicular to this, must have slope equal to 2.

Therefore, the (point-slope) equation of the line passing through P(1,3) is

$$y = 3 = 2(x - 1).$$

3. Sketch a graph of the function $f(x) = -(x+2)^2$. Don't forget to make a table of values for f.

We shall go over that in class: the necessary steps were to realize it is a parabola facing down, shifted two units to the left. Then, you would make a table of values, and make a sketch.

- 4. The graph of the functions f and g are given (f is in bold, g is dashed).
 - a) Which is larger, f(6) or g(6)? g(6).
 - b) Find the values of x for which f(x) = g(x).

The values are $\{2, 5, 7\}$. Note that I am using curly brackets, to emphasize I am only picking those values.

c) Find the values of x for which $f(x) \leq g(x)$.

Here, we are looking for a range of values, an interval of values: in this case, the answer is $[1,2] \cup [5,7]$ (closed brackets). Note that I am choosing interval notation, because there are a lot of numbers between 1 and 2, and between 5 and 7. Another way you could have answered is $\{x \in \mathbb{R} | 1 \le x \le 2 \text{ or } 5 \le x \le 7\}$.

d) Find the values of x for which f(x) > g(x).

Same discussion as above: the answer is $(2,5) \cup (7,8]$ (why do I have open brackets in all but the last bracket, on 8?). Another way you could have answered is $\{x \in \mathbb{R} | 2 < x < 5 \text{ or } 7 < x \leq 8\}$.

5. The graph of y = f(x) is given in bold. Match each equation with its graph, and explain your answer.

a) y = f(x - 4)

Graph is shifted right by 4 units.

b) y = f(x) + 3

Graph is shifted up by 3 units.

c) y = 2f(x+6)

Graph is stretched up by a factor of two, and then shifted left by 6 units.

d) y = -f(2x)

Graph is shrunk in the horizontal direction by a factor of 1/2, and is reflected across the x-axis.

Note: Almost all of you got this one right: we just need to work on how to better justify our answers.

6. Find the functions $f \circ g$ and $g \circ f$, and their domains, for $f(x) = \frac{2}{x}$ and $g(x) = \frac{x}{x+2}$.

$$f \circ g(x) = f(g(x)) = \frac{2}{\frac{x}{x+2}} = \frac{2x+4}{x}.$$

Domain: $\{x \in \mathbb{R} | x \neq -2 \text{ and } x \neq 0\} = (-\infty, -2) \cup (-2, 0) \cup (0, +\infty).$

Note that I also excluded x = -2, because if we tried to compute f(g(-2)), we would run into a problem, because g(-2) is not defined.

$$g \circ f(x) = g(f(x)) = \frac{2/x}{2/x+2} = (\frac{2}{x})(\frac{x}{2x+2}) = \frac{1}{x+1}$$

Domain: $\{x \in \mathbb{R} | x \neq -1 \text{ and } x \neq 0\}(-\infty, -1) \cup (-1, 0) \cup (0, +\infty).$

Note that I also excluded x = 0, because if we tried to compute g(f(0)), we would run into a problem, because f(0) is not defined.