Final Exam, Monday 21 December 2009
Math 308

Instructions: Please answer **nine** of the **eleven** problems. Write down the number of the problems you choose to skip in the space provided next to your name above. **Important: No books, calculators, or notes are allowed. Turn off cell phones, alarms, and anything else that makes noises!** You must show **all** your work to receive credit. Any crossed out work will be disregarded (even if correct). Write **one** clear answer with a coherent derivation for each question. Good luck!

[**1**] **(10 pts)** Is the compound statement $(P \vee (Q \Rightarrow R)) \vee (T \vee (\sim T))$ a tautology, a contradiction or neither?

[**1**] (10 pts)

Please leave blank!

[2] (10 pts) Write down the negations of each of the following statements.

(a) For all $x \in \mathbb{R}$, if $x > 2$ then $x^2 > 4$.

(b) There exist infinitely many odd integers n such that n and $n + 2$ are both prime.

(c) For all sets A and B, $A \cap B \subset A$.

(d) For all $\epsilon > 0$ there exists a postive integer N such that for all postive integers $n > N$, $\sin(1/n) < \epsilon$.

[**2**] (10 pts)

Please leave blank!

[3] (10 pts) Prove or disprove: for every positive integer n,

$$22n^7 + 26n^6 + 4n^4 + 18n^2 + 20n + 17$$

is prime.

[3] (10 pts)

Please leave blank!

[4] **(10 pts)** Let A and B be two nonempty sets such that $|A| < |B|$. Prove that there exists an injection from $\mathcal{P}(A)$ to $\mathcal{P}(B)$.

[**4**] (10 pts)

Please leave blank!

[5] (10 pts) Prove or disprove: For all natural numbers n, if $1 \leq n \leq 10$ and n is odd, then $n | 2^{n-1} - 1$.

[5] (10 pts)

Please leave blank!

[6] (10 pts) Prove: For $n > 1$,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + (n)(n+1) = (n)(n+1)(n+2)/3.$$

[**6**] (10 pts)

Please leave blank!

[7] (10 pts) Prove that for all positive integers $n \geq 2$,

$$\sum_{j=1}^{n} \frac{1}{\sqrt{j}} = 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$

[**7**] (10 pts)

Please leave blank!

[8] (10 pts) Let $\mathbb{I}$ be the set of irrational numbers. Define

$$f : \mathbb{I} \to \mathbb{I} \quad \text{by} \quad f(x) = \frac{3x}{x-2}.$$

Is f an injection? Is f a surjection? Justify your answers.

[8] (10 pts)

Please leave blank!

[9] (10 pts) The relation R on $\mathbb{Z}$ defined by $a\ R\ b$ if and only if $a^3 \equiv b^3 \pmod{13}$ is known to be an equivalence relation. Determine the distinct equivalence classes.

[**9**] (10 pts)

Please leave blank!

[10] (10 pts) Let $f(n) = \frac{1}{n} + \frac{(-1)^n}{n+1}$ and let S be the set

$$S = \{f(n) : n \in \mathbb{N}\}.$$

Does S have an infimum? If so, what is it? Does S have a supremum? If so, what is it?

[10] (10 pts)

Please leave blank!

[11] (10pts) Prove the *Archimedian Property:* If a and b are positive real numbers such that $a < b$, then there exists a natural number n such that $na > b$.

[**11**] (10 pts)

Please leave blank!