Final Exam, Monday, December 16, 2013
Math 308

Instructions: Please answer **nine** of the **eleven** problems. Write down the number of the problems you choose to skip in the space provided next to your name above. **Important: No books, calculators, or notes are allowed. Turn off cell phones, alarms, and anything else that makes noises!** You must show **all** your work to receive credit. Any crossed out work will be disregarded (even if correct). Write **one** clear answer with a coherent derivation for each question. Good luck!

Properties of real numbers to use in problem 9

A1. $a + (b + c) = (a + b) + c$ for all a, b, c.
A2. $a + b = b + a$ for all a, b.
A3. $a + 0 = a$ for all a.
A4. For each a, there is an element $-a$ such that $a + (-a) = 0$.
M1. $a(bc) = (ab)c$ for all a, b, c.
M2. $ab = ba$ for all a, b.
M3. $a \cdot 1 = a$ for all a.
M4. For each $a \neq 0$, there is an element a^{-1} such that $aa^{-1} = 1$.
DL. $a(b + c) = ab + ac$ for all a, b, c.
Thm1. $a + c = b + c$ if and only if $a = b$.
Thm2. $a \cdot 0 = 0$ for all a.

Name:

[1] (11 pts)

Let $a, b, n \in \mathbb{Z}$, where $n \geq 2$. Prove that if $a \equiv b \pmod{n}$, then $a^2 \equiv ab \pmod{n}$.

[2] **(11 pts)** Let A and B be sets. (We also assume implicitly that A and B are subsets of a universe U, but you can give a proof without referring to U.)

Prove that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.

[3] (11 pts) Prove or disprove: There exist odd integers a and b such that $4|a^2 + b^2$.

[4] **(12 pts)** The sequence $\{a_n\}$ is defined recursively by $a_1 = 2$, $a_2 = 6$, and for $n \geq 3$,

$$a_n = 2a_{n-1} - a_{n-2} + 2.$$

Prove that $a_n = n(n+1)$ for all $n \in \mathbb{N}$.

[5] **(11 pts)** Prove or disprove: Let A and B be sets. If $A - B = A$ and $B - A = B$, then $A \cap B = \emptyset$.

[**6**] **(11 pts)** Let R be the relation on $\mathbb{Z} - \{0\}$ defined by

$$a \; R \; b \text{ if and only if } ab > 0.$$

Prove that R is an equivalence relation.

[7] (11 pts) Let A, B, and C be nonempty sets, and let f, g, and h be functions such that $f : A \to B$, $g : B \to C$, and $h : B \to C$. Prove that if f is onto and $g \circ f = h \circ f$, then $g = h$.

[8] **(11 pts)** Let $A = \left\{x \in \mathbb{R} : x = 1 - \frac{1}{n} \text{ for some } n \in N\right\}$. Find $\sup A$ and prove your result is correct.

[9] (11 pts) Prove that $(-1)a = -a$ for any $a \in \mathbb{R}$. (The notation $-a$ represents the additive inverse of a. This means -1 represents the additive inverse of 1. Thus, there is something to prove here!)